ON A MIXED PROBLEM OF ELASTICITY THEORY FOR A WEDGE

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A plane mixed problem of elasticity theory for an infinite wedge is considered. There is a slit of finite length on the bisector of the wedge angle, and a normal loading of intensity $\sigma_\theta = -q(r)$ is applied to its surface.

The case when the relative distance μ between the slit and the wedge vertex is zero was investigated in [1 and 2]. A solution of the considered problem is found for large values of the parameter μ. The solution of the mentioned problem is expounded below for the whole range of variation of the parameter $0 \leqslant \mu < \infty$. A method whose idea is expounded in [4] is used in the solution. The method permits reduction of the problem to the determination of the function $\gamma(r)$ which describes the shape of the slit surface, from a Fredholm integral equation of the second kind. Moreover, in contrast to [1 and 2], an approximate solution of the problem is obtained here for the case $\mu = 0$ in the form of formulas of simple structure. The mathematical apparatus of the Wiener-Hopf method is hence used.

1. Formulation of the problem. In an elastic wedge bounded by the rays $\theta = \pm\alpha$ $(0 \leqslant r < \infty)$ let there be a slot occupying the domain : $\theta = 0$, $a \leqslant r \leqslant b$. The slot is kept in the open state by normal forces $q(r)$ applied to its surface. The following conditions may hence be satisfied on the wedge faces: (1) the wedge is squeezed between two smooth stiff bases, there are no friction forces between the bases and the wedge; (2) total adhesion occurs between the wedge and the stiff bases; (3) the faces of the wedge are stress-free. To determine the function $\gamma(r)$ and the coefficient of normal stress intensity N at $r = a$ and $r = b$ $(\theta = 0)$.

The problem can be reduced to determining $\gamma(r)$ from the integral equation [3]

$$\int_a^b \frac{\gamma(\rho)}{\rho} Q\left(\ln\frac{\rho}{r}\right)d\rho = -\frac{\pi}{\Delta}[p(r)+R], \quad Q(t) = \int_0^\infty L(u,\alpha)\sin(ut)\,du, \qquad \Delta = \frac{E}{2(1-\nu^2)}$$

$$(a \leqslant r \leqslant b) \qquad (1.1)$$

Here R is a constant determined in the solution of the equation, E is Young's modulus, ν the Poisson coefficient, and the function $p(r)$ is connected with $q(r)$ by means of the relationship

$$p(r) = \int q(r)dr \qquad (1.2)$$

For the conditions considered on the wedge faces the function $L(u, \alpha)$ is

$$(1) \qquad L(u,\alpha) = \frac{\operatorname{sh} 2u\alpha + u\sin 2\alpha}{\operatorname{ch} 2u\alpha - \cos 2\alpha} \qquad (1.3)$$

(2) $$L(u,\alpha)=\frac{\varkappa\,\mathrm{ch}\,2u\alpha+u^2(1-\cos 2\alpha)+0.5(1+\varkappa^2)}{\varkappa\,\mathrm{sh}\,2u\alpha-u\sin 2\alpha}\qquad(\varkappa=3-4\nu)$$

(3) $$L(u,\alpha)=2\,\frac{\mathrm{sh}^2\,u\alpha-u^2\sin^2\alpha}{\mathrm{sh}\,2u\alpha+u\sin 2\alpha}$$

Let us note the following properties of the function $L(u,\alpha)$:

$$\begin{gathered}L(u,\alpha)\to 1+O(e^{-2u\alpha})\ \text{ for }\ u\to\infty\quad(\text{for conditions (1), (2), (3)})\\ L(u,\alpha)\to c^{-1}\pi u+O(u^3)\ \text{ for }\ u\to 0\quad(\text{for conditions (1), (3)})\\ L(u,\alpha)\to(\pi u)^{-1}d+O(u)\ \text{ of }\ u\to 0\quad(\text{for condition (2)})\end{gathered}\tag{1.4}$$

$$\begin{gathered}c=\pi\,\frac{1-\cos 2\alpha}{2\alpha+\sin 2\alpha}\quad(\text{for condition (1)}),\\ c=\pi\,\frac{2\alpha+\sin 2\alpha}{2\alpha^2-2\sin^2\alpha}\quad(\text{for condition (3)}),\qquad d=\pi\,\frac{(1+\varkappa)^2}{4\varkappa\alpha-2\sin 2\alpha}\end{gathered}\tag{1.5}$$

The case $\alpha=\pi$ for conditions (1) on the wedge faces, which corresponds to the problem of a slot in a plane, will be henceforth excluded from consideration.

2. Solution of the problem for conditions (1) and (3) on the wedge faces. Let us approximate the function $L(u,\alpha)$ as follows:

$$L(u,\alpha)\approx\mathrm{th}\,\frac{\pi u}{c}+\sum_{i=1}^{M_1}A_i\,\frac{u^3}{\mathrm{ch}\,\dfrac{\pi u}{m_i}}\qquad(i=1,\ldots,M_1)\tag{2.1}$$

It is easy to see that the approximation (2.1) actually reflects the behavior of the function $L(u,\alpha)$ at zero and infinity for the considered conditions on the wedge faces. Let us present values of the quantities

$$\beta_j=\max\left[\frac{1}{L(u,\alpha)}\left|L(u,\alpha)-\mathrm{th}\,\frac{\pi u}{c}\right|\right]100\%$$

for conditions (1) ($j=1$) and (3) ($j=3$) on the wedge faces

$\alpha=$	30	45	60	75	90	105	120	135	150	165	180	(°)
$\beta_1=$	48	25	10	1.5	0	1	4	8	4	46	—	(%)
$\beta_3=$	>50	>50	42	18	2.5	4.5	4	2	0.5	0.05	0	(%).

Substituting $L(u,\alpha)$ in the form (2.1) into (1.1), and taking the inner integral, we find

$$c\int_a^b\frac{\gamma(\rho)\,\rho^{0.5c-1}}{\rho^c-r^c}\,d\rho=-\pi f(r)\qquad(a\leqslant r\leqslant b)\tag{2.2}$$

$$f(r)=\frac{1}{\Delta\sqrt{r^c}}\,[p(r)+R]+\int_a^b\gamma(\tau)\,\omega(\tau,r)\,d\tau\tag{2.3}$$

$$\omega(\tau,r)=\frac{1}{8\pi}\sum_{i=1}^{M_1}A_im_i^4\,\frac{r^{0.5(m_i-c)}(\tau^{m_i}-r^{m_i})}{\tau^{1-0.5m_i}(\tau^{m_i}+r^{m_i})^4}\,[20(r\tau)^{m_i}-(\tau^{m_i}-r^{m_i})^2]\tag{2.4}$$

Applying the inversion formula to (2.2), we obtain

$$\begin{gathered}\gamma(r)=\frac{c}{\pi}\sqrt{r^c(r^c-a^c)(b^c-r^c)}\left\{\frac{1}{\Delta}\int_a^b\frac{\sqrt{\rho^c}\,[p(\rho)+R]\,d\rho}{\rho\sqrt{(\rho^c-a^c)(b^c-\rho^c)}\,(\rho^c-r^c)}+\right.\\ \left.+\int_a^b\frac{\rho^{c-1}d\rho}{\sqrt{(\rho^c-a^c)(b^c-\rho^c)}\,(\rho^c-r^c)}\int_a^b\gamma(\tau)\,\omega(\tau,\rho)\,d\tau\right\}\end{gathered}\tag{2.5}$$

Here the boundedness condition of the function $\gamma(r)$

$$\int_a^b \frac{\rho^{c-1} f(\rho)\, d\rho}{\sqrt{(\rho^c - a^c)(b^c - \rho^c)}} = 0 \tag{2.6}$$

should be satisfied [6].

The integrals in (2.5) are understood in the Cauchy principal value sense. Moreover, the double integral in (2.5) admits an interchange in the order of integration since $\gamma(\tau)$ and $\rho^{c-1}[(\rho^c - a^c)(b^c - \rho^c)]^{-1/2}\, \omega(\tau, \rho)$ are integrable functions in $[a, b]$.

Substituting $f(\rho)$ in the form (2.3) into (2.6), and eliminating R from the obtained relationship and from (2.5), we obtain a Fredholm integral equation of the second kind for $\gamma(r)$

$$\gamma(r) = \frac{c}{\pi\Delta} \sqrt{r^c (r^c - a^c)(b^c - r^c)} \left[T_{-1}(r) + \int_a^b \gamma(\tau)\, \Omega_1(\tau, r)\, d\tau\right] + \\ + \frac{c\sqrt{b^c}}{\pi\Delta} \Lambda(r) \left[S_{-1} + \int_a^b \gamma(\tau)\, \Omega_2(\tau)\, d\tau\right] \tag{2.7}$$

$$T_{-1}(r) = \int_a^b \frac{\sqrt{\rho^c}\, p(\rho)\, d\rho}{\rho\sqrt{(\rho^c - a^c)(b^c - \rho^c)}\,(\rho^c - r^c)}, \qquad S_{-1} = \int_a^b \frac{\sqrt{\rho^c}\, p(\rho)\, d\rho}{\rho\sqrt{(\rho^c - a^c)(b^c - \rho^c)}}$$

$$\Omega_1(\tau, r) = \Delta \int_a^b \frac{\rho^{c-1}\omega(\tau, \rho)\, d\rho}{\sqrt{(\rho^c - a^c)(b^c - \rho^c)}\,(\rho^c - r^c)}, \qquad \Omega_2(\tau) = \Delta \int_a^b \frac{\rho^{c-1}\omega(\tau, \rho)\, d\rho}{\sqrt{(\rho^c - a^c)(b^c - \rho^c)}}$$

$$\Lambda(r) = \frac{F(\delta, k)}{K(k)} E(k) - E(\delta, k) + \frac{\sqrt{(r^c - a^c)(b^c - r^c)}}{\sqrt{r^c b^c}}$$

$$k = \sqrt{1 - \varepsilon^c}, \quad \varepsilon = a/b, \quad \delta = \arcsin\{(r^c - a^c)^{1/2}[r^c(1 - \varepsilon^c)]^{-1/2}\}$$

Here $F(\delta, k)$ and $E(\delta, k)$ are elliptic integrals of the first and second kinds, respectively; $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively.

We seek the solution (2.7) by successive approximations

$$\gamma(r) = \gamma_0(r) + \gamma_1(r) + \ldots + \gamma_n(r) + \ldots \tag{2.8}$$

As the first approximation we take an expression corresponding to the first member of the approximation (2.1). The zero and successive approximations are determined by means of Formulas

$$\gamma_n(r) = \frac{c}{\pi\Delta} \sqrt{r^c (r^c - a^c)(b^c - r^c)}\, T_{n-1}(r) + \frac{c\sqrt{b^c}}{\pi\Delta} \Lambda(r)\, S_{n-1} \quad (n = 0, 1, \ldots) \tag{2.9}$$

$$T_n(r) = \int_a^b \gamma_n(\tau)\, \Omega_1(\tau, r)\, d\tau, \quad S_n = \int_a^b \gamma_n(\tau)\, \Omega_2(\tau)\, d\tau \quad (n = 0, 1, \ldots)$$

For $r = a$ and $r = b$ the normal stress intensity coefficient N is determined from the following conditions:

$$N_a = \lim_{r \to a} \Delta \sqrt{r - a}\, \frac{d\gamma}{dr}, \qquad N_b = - \lim_{r \to b} \Delta \sqrt{b - r}\, \frac{d\gamma}{dr} \tag{2.10}$$

Substituting $\gamma(r)$ in the form (2.8) into (2.10) and taking account of (2.9), we obtain

$$N_a = \frac{c\sqrt{c}\sqrt{b^c}}{2\pi\sqrt{a}} \sum_{n=0}^{\infty} \left[a^c \sqrt{1 - \varepsilon^c}\, T_{n-1}(a) + \frac{E(k) - \varepsilon^c K(k)}{K(k)\sqrt{1 - \varepsilon^c}} S_{n-1}\right] \tag{2.11}$$

$$N_b = \frac{c\sqrt{c}\sqrt{b^c}}{2\pi\sqrt{b}}\sum_{n=0}^{\infty}\left[b^c\sqrt{1-\varepsilon^c}T_{n-1}(b) + \frac{K(k)-E(k)}{K(k)\sqrt{1-\varepsilon^c}}S_{n-1}\right] \quad (2.12)$$

3. Solution of the problem for the case when condition (2) is satisfied on the wedge faces. Let

$$L(u,\alpha) \approx \operatorname{cth}\frac{\pi u}{d} + \sum_{i=1}^{M_2} B_i u^2 \operatorname{csch}\frac{\pi u}{n_i} \qquad (i = 1, \ldots, M_2) \quad (3.1)$$

approximate the function $L(u, \alpha)$.

It is seen from (3.1), (1.4) and (1.5) that the approximation (3.1) truly reflects the behavior of the function $L(u, \alpha)$ at zero and infinity when condition (2) is satisfied on the wedge face . Values of the quantity

$$\beta_2 = \max\left[\frac{1}{L(u,\alpha)}\left|L(u,\alpha) - \operatorname{cth}\frac{\pi u}{d}\right|\right] 100\% \quad \text{for } \nu = 0.3$$

are presented below

$\alpha =$	15	30	45	60	75	90	105	120	135	150	165	180 (°)
$\beta_2 =$	10	5	7	10	10	8.5	5.5	3	1	0.5	1.5	1.5 (%)

Substituting $L(u, \alpha)$ in the form (3.1) into (1.1), we find

$$d\int_a^b \frac{\rho^{d-1}\gamma(\rho)\,d\rho}{\rho^d - r^d} = -\frac{\pi}{\Delta}[p(r) + H] - \frac{\pi}{\Delta}\int_a^b \gamma(\tau)\varphi(\tau, r)\,d\tau \quad (3.2)$$

$$\varphi(\tau, r) = \frac{\Delta}{\pi}\sum_{i=1}^{M_2} B_i n_i^3 \frac{(\tau r)^{n_i}(\tau^{n_i} - r^{n_i})}{\tau(\tau^{n_i} + r^{n_i})^3} \quad (3.3)$$

Applying the inversion formula to (3.2), we obtain a Fredholm integral equation of the second kind in $\gamma(r)$

$$\gamma(r) = \frac{d}{\pi\Delta}\sqrt{(r^d - a^d)(b^d - r^d)}\left[P_{-1}(r) + \int_a^b \gamma(\tau)\Phi(\tau, r)\,d\tau\right] \quad (3.4)$$

$$P_{-1}(r) = \int_a^b \frac{\rho^{d-1}p(\rho)\,d\rho}{\sqrt{(\rho^d - a^d)(b^d - \rho^d)}(\rho^d - r^d)}, \quad \Phi(\tau, r) = \int_a^b \frac{\rho^{d-1}\varphi(\tau, \rho)\,d\rho}{\sqrt{(\rho^d - a^d)(b^d - \rho^d)}(\rho^d - r^d)}$$

The integrals in (3.4) are understood in the Cauchy principal value sense. We seek the solution of (3.4) in the form (2.8). Furthermore, reasoning as in the solution of (2.7), we obtain

$$\gamma_n(r) = \frac{d}{\pi\Delta}\sqrt{(r^d - a^d)(b^d - r^d)}\,P_{n-1}(r), \qquad P_n(r) = \int_a^b \gamma_n(\tau)\Phi(\tau, r)\,d\tau \quad (3.5)$$

$$(n = 0, 1, \ldots)$$

$$N_a = \frac{d\sqrt{a^d b^d}\sqrt{\varepsilon^d(1-\varepsilon^d)}}{2\pi\sqrt{a}}\sum_{n=0}^{\infty}P_{n-1}(a), \qquad N_b = \frac{d\sqrt{a^d b^d}\sqrt{1-\varepsilon^d}}{2\pi\sqrt{b}}\sum_{n=0}^{\infty}P_{n-1}(b) \quad (3.6)$$

Let us note that the zero approximation of the solution of (3.2) corresponds to the first member of the approximation (3.1).

4. Solution of the problem for the case when the slot starts from the vertex of the wedge angle $(a = 0)$. Let us differentiate both sides of (1.1) with respect to r. Then integrating the obtained relationship by parts, we find

$$\int_0^b \gamma'(\rho)\, Q\left(\ln\frac{\rho}{r}\right) d\rho = -\frac{\pi}{\Delta} r q(r) \qquad (0 \leqslant r \leqslant b) \tag{4.1}$$

Let us make a change of variable in (4.1) by means of Formulas

$$\tau = \ln\ b/\rho, \quad t = \ln\ b/r \tag{4.2}$$

and let us introduce the notation

$$e^{-\tau}\gamma'(be^{-\tau}) = \psi(\tau), \quad i/\Delta e^{-t} q(be^{-t}) = w_+(t) \tag{4.3}$$

We hence obtain

$$i\int_0^\infty \psi(\tau)\, Q(\tau - t)\, d\tau = \pi w_+(t) \qquad (0 \leqslant t < \infty) \tag{4.4}$$

Let us apply a Fourier transformation in the variable t to (4.4)

$$\Psi_+(s) \quad L(s, \ \alpha) = W_+(s) + W_-(s) \tag{4.5}$$

Here $W_+(s)$ and $\Psi_+(s)$ are the Fourier transforms of the functions $w_+(t)$ and $\psi(t)$; $W_-(s)$ is the Fourier transform of the function corresponding to the stress originating in the wedge on its extension beyond the slot. Moreover, let us consider the case when conditions (1) and (3) are satisfied on the wedge faces. To carry out the factorization, let us represent the function L as follows [7]:

$$L(s, \alpha) = \frac{s\sqrt{s^2 + D_1^2}}{s^2 + E_1^2} H(s) \qquad \left(\frac{D_1}{E_1^2} = \frac{\pi}{c}\right) \tag{4.6}$$

It is seen from (4.6), (1.3) and (1.4) that the function $H(s)$ is regular in the strip $\Pi_1(-E_1 < \operatorname{Im} s < E_1)$ $(E_1 < D_1)$, is even in s, and positive on the real axis where $H(0) = 1$ and $H(s) = 1 + O(s^{-2})$ for $|s| \to \infty$ in the strip of regularity. It hence follows that the function $\chi(s) = \ln H(s)$ is also regular in the strip Π_1 and

$$\chi(s) = \chi_+(s) + \chi_-(s), \qquad \chi_\pm(s) = \pm\frac{1}{2\pi i}\int_{-\infty \mp i\tau_1}^{\infty \mp i\tau_1} \frac{\ln H(\zeta)}{\zeta - s}\, d\zeta \tag{4.7}$$

and $0 < \tau_1 < E_1$, the function $\chi_+(s)$ is regular in the half-plane $\operatorname{Im} s > -E_1$, the function $\chi_-(s)$ is regular in the half-plane $\operatorname{Im} s < E_1$. Taking account of (4.6), (4.7) we can write (4.5) as

$$\Psi_+(s) H_+(s) \frac{\sqrt{s + iD_1}}{s + iE_1} - g_+(s) = g_-(s) + \frac{W_-(s)(s - iE_1)}{H_-(s)\, s\sqrt{s - iD_1}} \tag{4.8}$$

$$g_+(s) = \frac{1}{2\pi i}\int_{-\infty + i\tau_1}^{\infty + i\tau_1} \frac{W_+(\zeta)(\zeta - iE_1)\, d\zeta}{\zeta H_-(\zeta)\sqrt{\zeta - iD_1}(\zeta - s)}, \quad g_-(s) = \frac{W_+(s)}{L_-(s, \alpha)} - g_+(s) \tag{4.9}$$

$$H_+(s) = \exp\chi_+(s), \qquad H_-(s) = \exp\chi_-(s) \tag{4.10}$$

Let $q(r) = q = \text{const}$. In this case

$$W_+(s) = -\frac{q}{\Delta\sqrt{2\pi}}\frac{1}{s + i} \tag{4.11}$$

Substituting (4.11) into (4.9) and utilizing residue theory, we find

$$g_+(s) = -\frac{q(1 + E_1)}{\Delta\sqrt{2\pi}\sqrt{-i}\sqrt{1 + D_1} H_-(-i)(i + s)} \tag{4.12}$$

It follows from (4.7) and (4.10) that

$$H_-(-i) = \exp\left\{\frac{1}{\pi}\int_0^\infty \frac{\ln H(z)\, dz}{1 + z^2}\right\} \tag{4.13}$$

Both sides of (4.8) coincide in the common strip of regularity Π_2 [sup $(-E_1, -1) <$ $< \operatorname{Jm} s < 0$] with some function $G(s)$ which is regular in the whole complex s plane. Since $\Psi(t) \sim 1/\sqrt{t}$ as $t \to +0$, then $\Psi_+(s) \sim 1/\sqrt{s}$ as $s \to \infty$ in the upper half-plane. Utilizing estimates of the functions $\Psi_+(s)$, $H_+(s)$ and $g_+(s)$ as $s \to \infty$, it can be shown that $G(s) \equiv 0$ by virtue of the Liouville theorem. Therefore

$$\Psi_+(s) = \frac{g_+(s)(s + iE_1)}{H_+(s)\sqrt{s + iD_1}} \tag{4.14}$$

We obtain the exact solution of the problem by finding the original of the function $\Psi_+(s)$. In order to obtain a solution suitable for practical usage, let us furthermore consider $H_+(s) \equiv 1$. This corresponds to the function $L(s, \alpha)$ being approximated by Expression

$$L(s, \alpha) \approx \frac{s\sqrt{s^2 + D_1^2}}{s^2 + E_1^2} \tag{4.15}$$

It can be proved that the error in the solution thus obtained will not exceed the error in approximating the function $L(u, \alpha)$ by (4.15). Finding the function $\psi(t)$, and returning to the old variables and notation, we finally obtain for cases (1) and (3)

$$\gamma(r) = \frac{qb(1 + E_1)}{\Delta H_-(-i)\sqrt{1 + D_1}}\left\{\frac{\sqrt{c}}{\sqrt{\pi}} \operatorname{erf}\sqrt{D_1 \ln(b/r)} + \frac{1 - E_1}{\sqrt{D_1 - 1}}\frac{r}{b} \operatorname{erf}\sqrt{(D_1 - 1)\ln(b/r)}\right\} \tag{4.16}$$

Substituting $\gamma(r)$ in the form (4.16) into the second relationship in (2.10), we find the exact value of the normal stress intensity coefficient N for $r = b$, $\theta = 0$

$$N_b = \frac{q\sqrt{b}(1 + E_1)}{\sqrt{\pi}\sqrt{1 + D_1}H_-(-i)} \tag{4.17}$$

Approximating the function $L(s, \alpha)$ by

$$L(s, \alpha) \approx \frac{s^2 + E_2^2}{s\sqrt{s^2 + D_2^2}}\left(\frac{E_2^2}{D_2} = \frac{d}{\pi}\right) \tag{4.18}$$

for the case when condition (2) is satisfied on the wedge faces, we obtain relationships to determine $\gamma(r)$ and N_b which are analogous to the relationships (4.16) and (4.17). Omitting intermediate calculations, we present the final expressions determining $\gamma(r)$ and N_b for the case $a = 0$ and condition (2) on the wedge faces

$$\gamma(r) = \frac{qr\sqrt{1 + D_2}}{\Delta H_-(-i)(1 - E_2^2)}\left\{\left(\frac{r}{b}\right)^{E_2 - 1}\sqrt{D_2 - E_2} \operatorname{erf}\left((D_2 - E_2)\ln\frac{b}{r}\right)^{1/2} - \right.$$
$$\left. - \sqrt{D_2 - 1} \operatorname{erf}\left((D_2 - 1)\ln\frac{b}{r}\right)^{1/2}\right\} \tag{4.19}$$

$$N_b = \frac{q\sqrt{b}\sqrt{1 + D_2}}{\sqrt{\pi}(1 + E_2)H_-(-i)} \tag{4.20}$$

Here $H_-(-i)$ is evaluated by means of (4.13), where $H(z)$ has the following form in this case:

$$H(z) = \frac{z\sqrt{z^2 + D_2^2}}{z^2 + E_2^2} L(z, \alpha) \tag{4.21}$$

5. Numerical investigation of the problem. Computations show that it is sufficient to limit oneself to a calculation of $\gamma_0(r)$ and $\gamma_1(r)$ for $0 \leqslant \beta_i \leqslant 50\%$ for any value of $\mu = 2 (\ln b/a)^{-1}$. In two cases: $\alpha = 1/2\,\pi$ and condition (1); $\alpha = \pi$ and condition (3), an exact solution of the problem is $\gamma_0(r)$. The maximum deviations of $\gamma_0(r)$ from the exact solution are obtained for $\mu \to 0$. As μ increases these deviations diminish, and $\gamma_0(r)$ tends to the exact solution of the appropriate problem of a slot in a plane as $\mu \to \infty$.

Let us present values of the quantities

$$\gamma^*(r) = \Delta(qb)^{-1}\gamma(r),\ N^*_a = 2(q\sqrt{2b})^{-1} N_a, \quad N^*_b = 2 \quad (q\sqrt{2b})^{-1}N_b,$$

evaluated for $q(r) = q = \text{const}$ ($p(r) = qr$) by means of the formulas obtained in Sections 2 and 3 (successive approximations), Section 4 (Wiener-Hopf method) herein, and by corresponding formulas of [3] (method of large μ)

a) condition (1)

$\gamma^*(\sqrt{ab})$	N_a^*	N_b^*	μ	
0.110	0.332	0.332	8	(zero approximation)
0.109	0.328	0.329	8	(method of large μ)
0.216	0.469	0.469	3.5	(zero approximation)
0.200	0.433	0.441	3.5	(first approximation)
0.202	0.435	0.443	3.5	(method of large μ)
0.636	—	0.900	0	(zero approximation)
0.553	—	0.785	0	(first approximation)
0.567	—	0.802	0	(Wiener-Hopf method)

The following values of the constants in approximations of the function $L(u,\alpha)$ were selected for $\alpha = 36.62°$ and condition (1): $A_1 = 4.9, A_2 = 0.82,\ m_1 = 1, m_2 = 1.5,$ $M_1 = 2,\ D_1 = 2.443,\ E_1 = 0.882.$ Here $H_-(-i) = 1.0089$, the error in the approximation (2.1) does not exceed 3%, the error in the approximation (4.15) 6.5%.

b) condition (2)

$\gamma^*(\sqrt{ab})$	$\gamma^*(^1/_2b)$	N_a^*	N_b^*	μ	
0.295	—	0.537	0.547	2	(zero approximation)
0.291	—	0.528	0.541	2	(method of large μ)
0	0.407	—	0.626	0	(zero approximation)
0	0.390	—	0.613	0	(first approximation
0	0.392	—	0.616	0	(Wiener-Hopf method)

c) condition (3)

$\gamma^*(\sqrt{ab})$	$\gamma^*(^1/_2b)$	N_a^*	N_b^*	μ	
0.326	—	0.607	0.590	2	(zero approximation
0.326	—	0.607	0.590	2	(method of large μ)
1.460	1.055	—	1.125	0	(zero approximation)
1.456	1.051	—	1.121	0	(Wiener-Hopf method)

The following values of the constants for approximations of the function $L(u,\ \alpha)$ were selected for $\alpha = 90°$ and conditions (2) and (3) on the wedge faces: $B_1 = 1,\ n_1 = 1,$ $M_2 = 1,\ E_2 = 0.832,\ D_2 = 1,\ A_i = 0\ (i = 1, 2,\ldots),\ D_1 = 2{,}549,\ E_1 = 1.652.$ Here $H_-(-i) = 0.9997$ for condition (2), $H_-(-i) = 1.0015$ for condition (3), the error in the approximation (3.1) does not exceed 0.5%, and the error in the approximation (4.15) and (4.18) does not exceed 3% and 1.5%.

In conclusion, let us note that the solution of the considered problem for small values of the parameter μ can be obtained by utilizing the method expounded in [8].

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CONTACT PROBLEM OF ELASTICITY THEORY FOR A HALF-STRIP

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The problem of the effect of a die on an elastic semi-infinite strip fixed rigidly along the short edge is considered. Integral equations for the contact pressure and normal stress at the clamping are formed. These equations are reduced to two systems of linear algebraic equations by the Bubnov-Galerkin method. Both systems turn out to be well specified, and their coefficient matrices are almost triangular.

Numerical computations were carried out for a die with a flat bottom, for an oblique and a parabolic die, and the high efficiency of the method was shown.

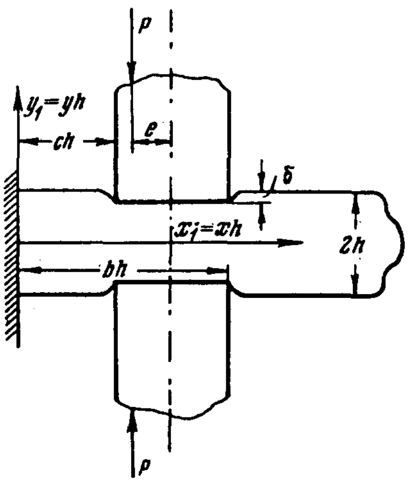

Fig. 1

1. Let us consider the problem of compressing a half-strip by two symmetrically disposed rigid dies under the following boundary conditions (Fig. 1):

$$u = v = 0, \quad x = x_1/h = 0$$

$$|y| = h^{-1}|y_1| \leqslant 1 \qquad (1.1)$$

$$\tau_{x_1y_1} = 0, \qquad y = \pm 1, \qquad 0 \leqslant x < \infty \quad (1.2)$$

$$\sigma_{y_1} = 0, \qquad y = \pm 1, \qquad 0 \leqslant x \leqslant c \qquad x \geqslant b \qquad (1.3)$$